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Effect of Ablation on Momentum Deposition in the Wake of a Re-Entry Vehicle

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The wake of an unpropelled vehicle is distinguished by a velocity and energy difference from the ambient condition. The vehicle loses momentum and energy, which are deposited in the wake behind it. For a slender, pointed re-entry vehicle, the momentum and energy loss from the vehicle caused by the pressure drag are spread out from the vehicle by the shock wave so that the changes in the energy and momentum of the affected flow are small, but the friction force causes a large change resulting in the part of the wake which is detectable.

For an ablating vehicle, the momentum deposited in the wake and the drag on the vehicle are not the same since the mass of the vehicle is changing. The momentum input to the wake is not the same for an ablating as for a non-ablating vehicle and is not equal to the drag on the vehicle if ablation is taking place. The purpose of this note is to explore this effect and determine the magnitude of the momentum loss associated with the ablation. The effect will be compared only with the friction drag, since this is the important drag for determining the wake of a sharp body, which is the case in which this phenomenon is of greatest interest.

The total loss of momentum of the vehicle I is the friction force F plus the momentum associated with the ablated mass $\dot{m}$:

$$I = F + \dot{m}V \tag{1}$$

The total mass loss by ablation is

$$\dot{m} = q/L = \tfrac{1}{2}\rho\, V^3 A C_H/gJL \tag{2}$$

where L is the heat of ablation of the material. The skin friction $F = \frac{1}{2}\,\rho V^2 A C_f$. Therefore,

$$I = \tfrac{1}{2}\,\rho A V^2 C_f\,[1 + (V^2 C_H/gJL\, C_f)] \tag{3}$$

Equation (3) shows that the actual skin-friction term is augmented by an ablation term, which depends on the ratio of freestream kinetic energy to the heat of ablation of the material and the ratio of heat transfer to friction coefficients.

Since the skin friction is affected by the rate of ablation, the preceding expression does not allow a comparison with the nonablation skin friction. To obtain this comparison, boundary-layer solutions, with fluid injection, must be considered. Because laminar flow over a cone lends itself to analytic treatment, with appropriate simplifying assumptions, this case will be used to demonstrate the important physical phenomena and the size of the effect. If the usual simplification of $Pr = Le = 1$ and $\rho\mu$ const are made, then the modified skin-friction parameter becomes a unique function of the modified blowing parameter similar to the incompressible case (Fig. 1).[1,2] By combining Reynolds analogy ($C_H = \frac{1}{2}C_f$, which follows from the assumptions already made) with Eq. (2), a relation between the blowing rate and skin friction based on the ablation properties can be found if all of the ablation products are considered to be emitted as gas:

$$\frac{(2\dot{m}/\rho VA)\ (Re_\infty/C)^{1/2}}{C_f(Re_\infty/C)^{1/2}} = \frac{V^2}{2gJL} \tag{4}$$

The relation is also plotted on Fig. 1 for different values of $V^2/2gJL$. The intersections of the curves are the solutions for C_f and $\dot{m}$. Equation (3) can now be written in terms of C_{f0}, the value of skin friction for a nonablating surface. This relation is plotted in Fig. 2. Typical values of L are between 1000 and 10,000 Btu/lb, so that at velocities of the order of 20,000 fps, $V^2/2gJL$ is between values like 1 and 10.

Although the simplified theory used here to describe the laminar boundary layer with injection is not precise, it does give a useful demonstration of the size of the effect being considered without obscuring the physical phenomena with involved numerical details. For a real case, appropriate values of Pr and Le should be used based on actual ablation products.

It may be concluded that the actual momentum deposited in the wake by the viscous effect of an ablative vehicle is larger than the nonablating vehicle. For laminar flow on a cone, it does not exceed approximately twice the nonablative vehicle value for the simplified case considered.

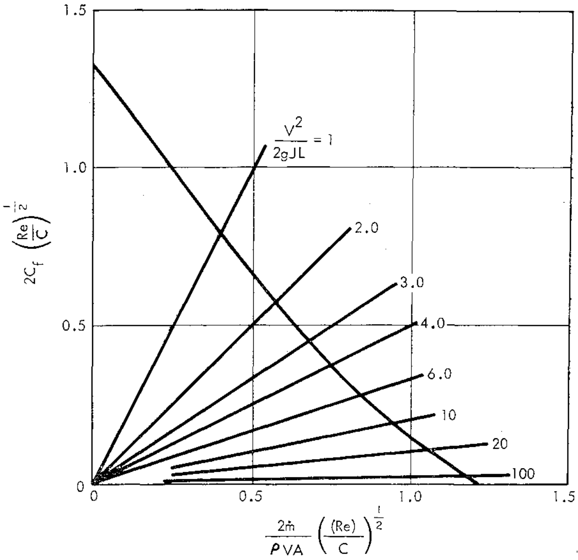

Fig. 1 Relation between skin friction and blowing rate.

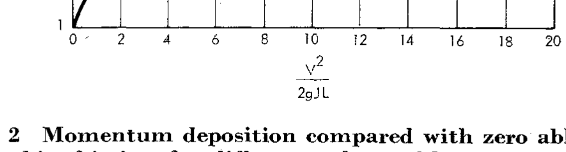

Fig. 2 Momentum deposition compared with zero ablation skin friction for different values of kinetic energy heat of ablation ratio.

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Similar Solutions for Merging Shear Flows II

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In Ref. 1 the merging of two infinite uniform shear flows behind a trailing edge was investigated, including the possible effects of the so-called "vorticity-induced" pressure gradient. The present note summarizes the results of subsequent numerical calculations. Detailed results covering a wide range of initial vorticity ratios of the two infinite shear flows are available in Ref. 2.

The notation of Ref. 1 is used. The plane incompressible flows are separated by a plane from $x = -\infty$ to $x = 0$; the initial vorticities are Ω_1 and $-\Omega_2$ in the upper and lower half-plane, respectively. The (laminar) merging takes place at $x > 0$. The similarity properties of the merging layer are accounted for by setting the stream function ψ equal to

$$\psi = (\Omega_1 \nu^2 x^2)^{1/3} f(\eta) \qquad (1)$$

where

$$\eta = y(\Omega_1/\nu x)^{1/3} \qquad (2)$$

as was first proposed by Goldstein.[3] Rott and Hakkinen[1] admitted the possibility of a vorticity-induced pressure gradient that was found to have the form

$$dp/dx = \rho\, c \Omega_1^{4/3} \nu^{2/3} x^{-1/3} \qquad (3)$$

where c is a dimensionless parameter to be discussed later.

These assumptions together with the standard boundary-layer formulation of the Navier-Stokes equations yield the total differential equation

$$3f''' + 2ff'' - f'^2 = 3c \qquad (4)$$

with boundary conditions $f'' = 1$ at $\eta = \infty$, and $f'' = -\Omega_2/\Omega_1 = -\beta$ at $\eta = -\infty$. The following behavior is then implied for the asymptotic velocity profiles as $\eta \to \pm\infty$:

$$f'_{+\infty} = \eta - l_1 \qquad f'_{-\infty} = -\beta(\eta - l_2) \qquad (5)$$

where l_1 and l_2 can be determined only after numerical integration of Eq. (4).

There are two families of solutions for which numerical calculations were performed. The first case is characterized by $c = 0$, i.e., vanishing pressure gradient, and the second one by unshifted asymptotic velocity profiles, i.e., $l_1 = l_2 = 0$, corresponding to a specific induced pressure gradient parameter $c \neq 0$. In terms of physical velocities, the general asymptotic behavior is

$$u_{+\infty} = \Omega_1[y - \Omega_1^{-1/3}(\nu x)^{1/3} l_1]$$
$$u_{-\infty} = \Omega_2[y - \Omega_1^{-1/3}(\nu x)^{1/3} l_2] \qquad (6)$$

To obtain the special solution with $l_1 = l_2 = 0$, the parameter c has to be varied until $l_1 - l_2 = 0$. Noting that Eq. (4) is invariant with respect to a change in the origin of η, it is then possible to find a position where the asymptotic u-distributions remain unshifted both for $y = +\infty$ and $y = -\infty$.

The order of magnitude of the shift as seen from Eq. (6) (noting that l_1 and l_2 are of order 1) gets properly small in the limit of vanishing kinematic viscosities but shows an undue growth for large x. The significance of this behavior is hard to ascertain, as the undisturbed velocity also grows without bounds for large $|\eta|$. Nevertheless, in a similar situation involving a flat-plate boundary layer in an infinite shear flow, Li[4] has first proposed that solutions involving a shift at infinity are not permissible.

The investigation of Murray[5] upheld Li's contention, provided that the shear flow is indeed extending to infinity laterally. In what limited sense only can a laterally bounded shear flow region be approximated by the infinite case has been shown by Toomre and Rott.[6] Infinite shear flow turned out to be a limit of questionable properties, both mathematically and physically. Only for a region whose extent is small compared to the lateral dimension of the oncoming shear flow can the Li-Murray solution be applied.

Similar investigations for the problem of merging shear flows are still incomplete. Here, the Li-Murray boundary conditions are applied without further discussion to the numerical calculation of the symmetric case, i.e., $\Omega_1 = -\Omega_2$, $l_1 = l_2 = 0$, and the corresponding value of the pressure gradient parameter turns out to be $c = 0.4089$. The remaining discussion is concerned with the possible significance of this solution for the merging of two identical Blasius boundary layers at the sharp trailing edge of a flat plate. The vorticity there, for a plate of the length L, is given by the Blasius solution and has the value

$$\Omega_1 = -\Omega_2 = \Omega = 0.332\, U_\infty^{3/2}(\nu L)^{-1/2} \qquad (7)$$

A quantity of special interest is the velocity at the line of symmetry $\eta = 0$:

$$u_0/U_\infty = 0.4795\, f'(0)(x/L)^{1/3} \qquad (8)$$

that is, for case 1, $c = 0$, Goldstein's leading term:

$$u_0/U_\infty = 0.772(x/L)^{1/3}$$

and for case 2, $l_1 = l_2 = 0$, present solution

$$u_0/U_\infty = 0.431(x/L)^{1/3}$$

How far downstream the new solution may be valid is not known precisely, but the analogy with the forementioned results of Toomre and Rott[6] permits a qualitative statement. Noticeable influence of the vorticity-induced pressure gradient can be found only in a region with streamwise dimension comparable to the lateral extent of the shear layer. The latter quantity is, for the present application, equal to the Blasius boundary-layer thickness:

$$\delta_{Bl}/L \cong 5(\nu/U_\infty L)^{1/2} = 5Re_L^{1/2} \qquad (9)$$

There exists a second limitation on the validity of the solution considered here; for regions too near to the trailing edge, the boundary-layer assumptions have to be abandoned in favor of new approximations of the Stokes or Oseen type.

For regions far from the trailing edge, an asymptotic expansion procedure, in which the similarity solution appears as the leading member, has been carried out by Hakkinen and O'Neil.[7] These expansions all diverge as the trailing edge is approached.

The main results of Hakkinen and O'Neil[7] can be summarized as follows: 1) the downstream centerline velocity is given by the series:

$$u_0 = \Omega^{2/3}\nu^{1/3}c^{1/3}(0.899 - 0.165[x(\Omega/\nu)^{1/2}]^{-2/3} - 0.885[x(\Omega/\nu)^{1/2}]^{-4/3} + \ldots) \qquad (10)$$

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